Motivation: global optimizer − prototype algorithm

Optimization problems occur in quite different settings. In politics, for instance in the context of sustainability, we nowadays discuss about welfare maximization under social, environmental and natural constraints. More down to earth, in statistics optimization oftentimes relates to functions (e.g. likelihood or least square functions) defined over some underlying parameter space while in engineering optimization occurs for instance when approximating functions (such as signals) by Fourier series or wavelets which involves estimation of the respective coefficients. In turn, bio-statistical applications may try to explore the optimal effect of some drug or substance constrained by potential undesirable side effects to patients.

Considering the variety of optimization problems that can occur there is an according choice of optimization tool boxes available in mathematical tools such as Mathematica or Matlab. However, choosing the best optimization algorithm for a given non-linear problem, such as considered in this project, may amount to a challenge even for full-fledged optimizers. In particular, derivative based methods are sensitive to the choice of a starting vector; the latter needs to be selected with care in order to make sure that solutions found are not just optimal locally but globally. In the latter respect so-called genetic algorithms have become popular not at least because they are less sensitive on starting values.

The present algorithm joins by some original means elements and ideas such as used in

- genetic algorithms (population approach);
- the simplex algorithm by Nelder and Mead (has nothing to do with the Simplex method employed to linear problems);
- Monte Carlo methods subject to variance reduction techniques (Las Vegas method);
- rejection methods such a simulated annealing (Metropolis-Hastings algorithm);
- classic downhill methods in regions where the objective function is differentiable (or at least can be approximated satisfactorily by differentiable functions).

In a first instance this new algorithm is supposed to be used for scrutinizing solutions found by established algorithms of choice to existing optimization problems and to analyze the cost (inherent to the underlying problem) of potentially not having found the globally optimal solution. The latter type of analysis will be crucial in order to convey the cost-benefit aspect to choosing the best optimization technique – presumably quite an unexplored area of research as of date. The latter type of study will be submitted to the <<project fair>> in the course of 2008.