1 Maximum Likelihood estimates of censored loss data

This project was carried out by Harry Niederau in collaboration with PhD student of the biophysical institute at the University of Zurich who had dealt with "multiple-mode" response surfaces within his doctoral research.

1.1 Motivation

Loss distributions in industrial insurance, or reinsurance in particular, are difficult to calibrate when the underlying statistical material (such as losses) is censored at various levels. Oftentimes, reinsurers only have loss information above retention levels of their insureds. Since retention levels typically vary along insurance cycles, this means that reinsurers have loss information without a homogenous reference deductible over time. In particular they have no loss information at all below the lowest self-retention ever. The academic and admittedly elegant solution to this problem is called EVT (extreme value theory). In essence this theory says that you only need know the limit distribution that the underlying distribution (which you assume for your population) is supposed to converge to if the self-retention grows arbitrarily large. Then, given you have enough statistical material left above this large threshold, you can try to reasonably fit the parameters of the limit distribution. This approach is typically applied to the reinsurance of catastrophic losses with a retuning period of 100 years and more. However this academic approach is not of much help for insurance layers between frequent (say annual) losses and losses with a returning period of 20 to 50 years say. This is typically where primary insurers write their business.

Relating back to the above sketched problem of parameter inference when losses have inhomogenous retention levels one can use conditional maximum likelihood techniques. Depending of course of the chosen distribution function the surfaces of such likelihood functions can be "tricky" in the sense that it may have various locally optimal points which can make finding the global optimum by means of classical downhill methods a lottery.

1.2 Approach

The Simplex method of Nelder and Mead was amended in a few respects. On the one hand we developed special rules for tuning the Simplex parameters whenever the Simplex would overshoot the allowed parameter ranges to make sure that the simplex would not collapse in the neighborhood of parameter limits. On the other hand we introduced random searches with slight variations of a fixed tuning parameter before ever a reflection was rejected and a contraction (in particular multidimensional) was performed. Also here the goal was to prevent the simplex from contracting too rapidly. Finally we amended the termination criteria which significantly enhanced stability in convergence.

1.3 Results

We could demonstrate with a couple of actual and censored loss data sets that parameter estimates we got from the Simplex optimization clearly outperformed (random) solutions found by standard Newton-Raphson type of techniques. Also we could demonstrate that whether finding the globally optimal solution or not does matter from a business perspective. We could sometimes observe variation of 50% and more on a pure loss expectation level depending on whether the global optimum was found or not. Needless to add that deviations for higher moments where even more significant.

1.4 (Possible) Conclusion

A prudent insurer may not discuss so much about questions like: should we apply a 15 or 20% standard deviation loading to our expected loss level. Rather one may pay attention to questions like: how do we make sure by appropriate estimation techniques not to underestimate mean and standard deviation by 50% and more in the first place.